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Aligning Asset Pricing Models and Neural Networks for Predicting Portfolio Returns in Frontier Markets

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Abstract: *Forecasting Portfolio returns is a challenging task, and conventional forecasting models have partially succeeded in dealing with the nonlinear and complex nature of Equity Markets. Artificial neural networks are a mathematical modeling approach that is resilient enough to forecast portfolio returns in volatile and nonvolatile markets and act like the human brain to simulate the behavior of stock prices. This research documents the predictive ability of Artificial Neural Networks (ANN) by using the constructs of Fama and French three-factor and five-factor models. A comprehensive methodology of neural networks is applied to achieve the purpose of forecasting. This methodology includes the declaration of the internal layers, the hidden layer neurons, and varying parameters for an effective ANN system. A rolling window scheme is applied to forecast the errors among the competing asset pricing models. The predictive performance of ANN is measured by the metric of mean squared error, and the accuracy of ANNs under both pricing models is evaluated by the Diebold Mariano test. The significant findings of the study include the identification of the optimum architecture of the ANN under both asset pricing models, the non-overfitting phenomenon of the networks, and the abnormal returns for the investors for holding high-risk portfolios.*

Key Words: Asset Pricing, Financial Forecasting, Portfolio Choice, Artificial Neural Networks, Financial Markets

JEL Classification: C45, D53, E37, G11, G17

Introduction

Equity markets spur important economic indicators and play a central role in shaping economies (Bonfiglioli, 2008). The large-scale investments in equity markets worldwide show the interest of investors from all quarters of society, which helps economies cultivate a strong base for economic development (Bekaert et al., 2005; Levine, 2008). However, the analysis of the returns earned by the investors in equity markets reveals that their average returns are low compared to the market (Malkiel, 2011).

This rate of return in bearish markets is even lower than the market returns. The reasons behind this non-synchronization in returns are beyond the understanding of typical investors. Some notable authors (Fama & French, 2015; Karaban & Maguire, 2012) disagree with this notion and contend that investors' returns, in some cases, are above the market in those same markets and situations. Behaviorists attribute the responsibility of variable returns to the panic and irrational decision-making of investors while investing in risky assets. Other reasons include the lack of the use of nonlinear techniques for forecasting, the application of wide-ranging financial and technical variables without providing a rationale from the established theory of asset pricing, and the lack of interest of the investigators to examine the forecasting techniques in the actual market environments (Gray et al., 2012).

Researchers from multiple disciplines have developed different linear and nonlinear techniques to forecast stock prices over the years. The application of linear predictive models in equity markets is simple,

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but they suffer from structural limitations in capturing the nonlinear nature of stock market data (Ang & Bekaert, 2007). Campbell and Shiller (1988) note that there is a nonlinear relationship between predictor variables and long-term stock returns, and the application of linear regression might produce spurious results.

The basic forecasting model of stock returns is the capital asset pricing model, which suggests that stock returns are related to market risk. This model is based on the mean-variance relationship of portfolio selection (Markowitz, 1952) and the equilibrium model of Tobin (1958) to predict asset prices. Ahmed and Javid (2008) have investigated the predictive performance of single factor and multifactor CAPM in conditional and unconditional form, and their findings suggest that the conditional asset pricing models show a significant improvement in predictability.

The application of nonlinear methods is a recent phenomenon in financial modeling, and these techniques have demonstrated 93% accuracy in forecasting stock returns. A study by Kanas and Yannopoulos (2001) explains that nonlinear predictions are significantly more precise than linear projections. The financial modeling of returns can be termed the mathematical representation of the thinking of investors (Sargent, 1993). The key stakeholders in financial markets have access to all the necessary information, but they still have to learn the optimal and profitable decision-making on the floor, and artificial neural network-based modeling possesses this ability to mimic human psychology (Guresen et al., 2011).

The mechanism of artificial neural networks bears a close resemblance to the human brain and can be used as a replacement for investors' decision making (Pettengill et al., 2012). The basic strength of the ANN system is that it can capture the last-moment changes in the stocks-related variables with greater accuracy, which can magnify the value of investors' decision-making in stock markets (Garson, 1998).

Several recent studies (Carvalhal & Ribeiro, 2008; Dunis et al., 2011; Fadlalla & Amani, 2014; Maknickienė & Maknickas, 2013; Qiu & Song, 2016) find that ANNs return better forecasting results than classical techniques, and the prediction performance of ANNs not only exceeds the conventional linear and nonlinear modelling but the empirical results of neural network systems are better and robust.

Applying ANN in finance has several challenges because an in-depth application of ANN entails a greater understanding of the basic theories of Physics, Computer Science, Artificial Intelligence, and Mathematics. In addition to these limitations and hindrances, most investigations find ANN as a better replacement for human wisdom in stock markets.

The broad horizons of ANNs include the data distribution, the number of neurons, and the application of various training functions under rolling window schemes. The time series of returns, measured by the composite variables of three factors and five factors CAPM, is processed with the help of the ANN algorithm in Pakistan's Stock Exchange, and the success or failure of the models is assessed with the help of t-statistics and the Diebold Mariano accuracy test. This study adds a substantial contribution to the theory of asset pricing by presenting a solution to the problem mentioned below.

"Can the Artificial Neural Networks methodology be applied to the asset pricing models to forecast stock returns and generate a rate of returns for the investors which is above the market"?

Some hallmark papers (Cao et al., 2005, 2011; Jan & Ayub, 2019; Olson & Mossman, 2003; Stansell & Eakins, 2004) describe the hybridization of asset pricing models and their composite variables with ANN. The principal finding of these studies is that the ANN can successfully separate the value stocks from other stocks, classify the securities into high and low earning returns, select the portfolios that can beat the market, and choose the best forecasting composite factors of the asset pricing models. These studies document some limitations, generating a research gap for the present study.

First, some forecasting studies cannot ascertain future results, and the element of uncertainty is a significant limitation of such research. Second, the researchers sometimes select the variables for which no past data are available, or the association between the selected variables is not justified by finance theory. Third, a comparative analysis of various asset pricing models is not conducted, and finally, these findings are based on the limited parameters of neural networks. The present study is based on the most efficient training techniques, which require a minimum time to generate minimum error between the actual and predicted values. This technique is also known as the damped least square method and presents solutions to nonlinear problems in finance. Other differentiating points of our study include the use of Portfolio returns, all possible combinations of ANN parameters, and the use of tailored computer coding instead of commercial software.

In section 1, we discuss the literature on the fundamental aspects and forecasting ability of ANNs in equity markets. Section 2 elaborates on the data sources, period of the study, the significant characteristics of the Pakistan Stock Exchange, and the formation of the portfolios of the Fama and French three- and five-factor CAPM. The model building of the ANN architecture and multifactor asset pricing models are also discussed in this section. The results of ANN-based capital asset pricing models are presented in section 3, along with the accuracy test results of the Diebold Mariano and t-statistics. Section 4 analyses the results, and conclusions are presented. Future research directions on further application of ANNs in another subdiscipline of finance and asset pricing theory are also presented in the Conclusion section.

Literature Review

Estimating the required return of investors can be termed a mathematical representation of the thinking of investors (Gunn & MacDonald, 2006). The investors interpret the rise and fall in the market prices of the stocks in different ways and express their sentiments in their buying and selling decisions. This reaction shows the continuous learning and adjustment of investors' behavior, and financial modeling in stock markets is based on this psychology of stakeholders (Cao et al., 2011; Sargent, 1993).

The principles of artificial neural networks draw their foundation from human thinking capability for approximating stakeholders' decision-making in a particular situation. The scientific modeling of human understanding in such a way has given a paradigm shift to the decision-making and industrial processes of present-day enterprises (Tkáč & Verner, 2016). The design of efficient algorithms for the NN system in forecasting is an active research topic because it demonstrates an excellent capacity to emulate the unexplored relationship between dependent and independent variables of noisy and nonlinear environments (Franses & Van Dijk, 2000). Econometricians consider this a modified version of the STAR family Techniques (Smooth Transition Autoregressive).

The application of artificial neural networks in the last two decades in almost all the branches of finance has given new hope to the forecasting of stock returns. Notable studies of artificial neural networks in finance are found in (Dunis et al., 2011; Guresen et al., 2011 Pettengill et al., 2012; Vanstone & Finnie, 2006). Most of these studies concentrate on the indices of stock markets or employ the financial variables of stock market activity, trading volume, dividends, accounting ratios, foreign exchange rates, and other financial variables. However, the selection of these variables ignores the established factors used and appreciated by stock market researchers.

A comparative analysis of ANN and other traditional techniques to assess the predictive performance of both these models documents that the system poses a tough challenge for conventional econometric techniques. An earlier study (Olson & Mossman, 2003) is considered a hallmark of ANN application in the securities market and probably the first application of ANN and asset pricing factors. Another study evaluates the predictive ability of random walk, neural networks, and linear autoregressive models and suggests that NN outperforms all other conventional models in forecasting.

Carvalhal and Ribeiro (2008) compare the predictive ability of ANN with three traditional forecasting techniques, and the findings provide substantial evidence that ANN is a better technique for predicting the indices. Dunis et al. (2011) present a comparative analysis of ANN and other traditional techniques to asses the prédictive performance of both models in the Athens stock markets. The study constructs various autoregressive technical variables as inputs to the network, and the findings recommend the adoption of the neural network by fund managers to enhance their returns in volatile markets. Many scholars have applied ANNs to Pakistan's equity, including (Fatima & Hussain, 2008 Haider & Nishat, 2009 Iqbal, 2013). These studies concentrate on the stock market index, and the findings suggest that ANN has successfully predicted the returns with high accuracy.

The findings of some important articles provide significant guidelines for the present study; for example, Coupelon (2007) documents the necessary guidelines in modeling the neural networks to represent the datasets correctly and present a good forecast. Cao et al. (2011) utilize the variables of beta, market cap, and book-to-market ratio to gauge the performance of both the linear and nonlinear models through various performance parameters. The study utilizes the feed-forward neural network design to determine the network's minimum error because 90% of the prediction studies in finance employ this architecture. Similarly, (Fadlalla & Amani, 2014; Hall & St. John, 1994; Rechenthin, 2014) provide valuable guidelines related to the selection of training methods, dataset distribution, activation methods, and other parameters of the NN system for our study.

It should be noted, however, that the implementation of ANN in forecasting does not consistently fetch better results, and this technique's inherent drawbacks sometimes hinder its application. The black box nature of the NN system produces the problem of overfitting the data. The overfitted model is not useful for any practical purpose in financial markets (Franses & Van Dijk, 2000; Rechenthin, 2014). These practical difficulties force researchers to believe that simple linear forecasting models better predict their desired goals. Some recent studies, i.e., (Fescioglu-Unver & Tanyeri, 2013; Stansell & Eakins, 2004; Vortelinos, 2017), find unfavourable results of ANN in forecasting compared to traditional techniques.

Data and Methodology

We aim to estimate the predictive utility of various asset pricing models in the presence of artificial neural networks using Pakistan's Stock Exchange (PSX) as the unit of analysis. The sampled firms include manufacturing companies listed on the KSE-100 following Fama and French (Fama & French, 2015). The selection criteria for the remaining firms from various sectors are adopted from (Ahmed & Javid, 2008). This criterion states that 1) the selected stock must be listed at KSE, 2) the data of the monthly price index and the book value, market equity, total assets, profitability, and volume traded by the sample companies must be available, and 3) the selected stocks must be traded for more than 90% of trading days during the study period.

We employ the data from January 2006 to December 20222. The data sources include the Central Bank of the Country and the DataStream of Thompson Reuters. This research employs monthly portfolio returns as the target or predicted variable based on listed firms on the Pakistan Stock Exchange. The selection of monthly data instead of weekly or daily data is that the study follows the standard method of Fama and French (1993) for the formation and selection of target returns.

The KSE-100 index is used to calculate market returns (Ayub et al., 2015). The stock prices and value of the index are used to calculate the returns, and these returns are normalized by applying the logarithmic returns without including the dividends because the market activity makes it a part of the stock prices, according to Ahmed and Javid (2008). We form three categories of high-, mid-, and low-risk portfolios and divide them into 30 portfolios. The descriptive statistics of the monthly returns of these portfolios (on an average basis) are presented in Table 1.

The price-related data, including opening, high, low, and closing prices, are obtained from the database of the Pakistan Stock Exchange. Various factors of the FF3F and FF5F models require the fundamental variables of market cap, B/M ratio, change in total assets, earnings, and EBIT to equity. Market capitalization is used to calculate the size factor, better known as the SMB factor. The second grouping of the firms is based on the ratio of book to market value of the firm's equity. This grouping results in the HML factor.

Table 1

Descriptive Statistics of High, Mid, and Low Beta Portfolios Returns (180 Monthly Counts)

Portfolio Type	Mean %	Median %	Minimum %	Maximum %	Std. Dev %	Var.%	Kurtosis	Skewness
High Beta (P1-P10)	0.21	0.39	-47.49	21.8	8.31	0.69	6.2407	-1.2788
Mid Beta (P11-P20)	0.29	0.38	-46.6	26.97	9.13	0.83	4.092	-0.855
Low Beta (P21-30)	0.52	0.52	-30.81	33.06	9.28	0.86	1.5822	-0.2291

We sort the firms by their respective market capitalization from low to high-ranking order. The first 50% of the group represents those firms whose market capitalization is low and are considered small (S) firms, while the second 50% describes large firms (B). The second sort is based on high and low B/M ratios. The firms are now placed into three groups. The first group is 30% of the sample and consists of those firms whose book-to-market ratio is high (B) and market capitalization is low (S). The second cluster is 40% of the sample and consists of firms having a medium B/M ratio (M) and small capitalization(s).

The last group is the remaining 30% of the sample, and their grouping is based on small-capitalization firms with a low B/M ratio. This procedure results in forming S/H, S/M, and S/L portfolios. The same procedure is applied to firms with high market capitalization, and large capitalization companies (B) are again sorted based on their high book-to-market ratio (H) and low book-to-market ratio (L). They are categorized into three groups, and applying the 30%, 40%, and 30% criteria of portfolios results in portfolios called B/H, B/M, and B/L. These six portfolios are now used to form the SMB factor for each year from 2006 to 2020.

Likewise, we sort again for investment factors from conservative to aggressive stocks, forming three categories of aggressive (A), medium (N), and conservative (C) investment stocks. These are distributed based on 30%, 40%, and 30% conventions to generate six portfolios as S/C, S/N, S/A, B/C, B/N, and B/A. Finally, we sort the size groups again using the same procedure to calculate the factor of profitability as weak (W), medium (N), and robust (R) stocks yielding S/R, S/N, S/W, B/R, B/N, and B/W portfolios.

The SMB factor (small minus large stocks) is calculated as follows:

$$SMB = \frac{(SH + SN + SL) - (BH + BN + BL)}{3} \dots\dots\dots\dots (1)$$

The HML factor (high minus low-value stocks) is calculated as follows:

$$HML = \frac{(SH + BH) - (SL + BL)}{2} \dots\dots\dots.. (2)$$

The CMA (conservative minus aggressive stocks) is calculated as follows:

$$CMA = \frac{(SC + BC) - (SC + BA)}{2} \dots\dots\dots\dots (3)$$

The RMW is calculated as robust minus weak stock:

$$RMW = \frac{(SR + BR) - (SW + BR)}{2} \dots\dots\dots\dots (4)$$

The FF3 CAPM and FF5 CAPM are the two models to be estimated and given, respectively, as follows:

$$R_P = R_f + \beta_1(R_{PSX} - R_f) + \beta_2(R_{SMB}) + \beta_3(R_{HML}) + \varepsilon_P \ \dots\dots. (5)$$

$$R_P = R_f + \beta_1(R_{PSX} - R_f) + \beta_2(R_{SMB}) + \beta_3(R_{HML}) + \beta_4(R_{CMA}) + \beta_5(R_{RMW}) + \varepsilon_P \ \dots..(6)$$

Aligning ANN and Asset Pricing Models

The back propagation training method has been implemented in this study, and the ANN system consists of three layers: the input, hidden, and output layers. The common practice documented in a study (Franses & Van Dijk, 2000) is that the architecture of the networks is determined by the hidden layers, although some studies declare the input and output layers as a part of the NN structure (Pyo et al., 2017). The neurons in the hidden and other layers are connected through a system of weights that calculate their output through mathematical functions. The interconnection of the neurons stores knowledge of the processing mechanism, giving rise to the network's short- and long-memory features.

The primary parameters of the system include first, the normalization, and preparation of the dataset in a particular tabular form—second, the training algorithm and, finally, the network architecture. The organization of the neural network system is a function of the neurodynamic and architecture, and the guidance provided by a study (Kaastra & Boyd, 1996) is worth noting. The details are given below.

In step 1, the target independent (input) and dependent (output) variables are declared for the NN system. The composite factors of FF3F and FF5F CAPM are applied as the inputs, and the portfolio returns are used as the output (target) for the NN system. The three- and five-factor models are decoded into ANN according to the following equation:

$$R_P = G\left(\alpha + \sum_{j=1}^{h} \alpha_j\right) + F\left(\beta_{0j} + \beta_{1j}(R_{PSX} - R_f)\right) + \beta_{2j}(R_{SMB}) + \beta_{3j}(R_{HML}) \ \dots.. (7)$$

$$R_P = G(\alpha + \sum_{j=1}^{h} \alpha_j) + F\left(\beta_{0j} + \beta_{1j}(R_{PSX} - R_f)\right) + \beta_{2j}(R_{SMB}) + \beta_{3j}(R_{HML}) + \beta_{4j}(R_{CMW}) + \beta_{5j}(R_{RWA}) \ldots\ldots (8)$$

In these equations, G (.) is the nonlinear activation function used in the hidden layer, and F (.) is the linear activation function of the output layer.

In step 2, the preprocessing of the data for the NN system is followed, which helps the NN system learn and store the significant relationships between the dependent and independent variables. The system uses its standards of normalizing the data between the upper and lower limits (Jasic & Wood, 2004), and we apply the sigmoid function to normalize the data between 0 and 1.

A common practice in traditional forecasting modeling is dividing the target data into in-sample and out-sample datasets. The equivalent terms for these datasets in the ANN system are training and testing datasets. The additional validation step is applied explicitly by the neural networks to be trapped in local minima, thus controlling the asymptotic nature of the network architecture identified in a study (2017). The testing data range from 5% to 20%, while the training dataset ranges between 60% and 90%, following the established convention of ANNs. The system's generalization (out-of-sample) feature is evaluated by testing the dataset after the training is completed.

This study identifies the optimal architecture of the ANN for the three- and five-factor models. We apply wide-ranging combinations of (training, testing, and validation) datasets and portfolio categories based on the risk level and the number of neurons. We apply 60-20-20 data combinations for training, validation, and training, and the program assigns a five percent increment to the dataset. The program generates 16 data combinations, and the details are given in Table 2.

The transfer function of ANN, also called activation, transformation, or squashing functions, calculates the output of a hidden layer (Kaastra & Boyd, 1996). We use the logistic (sigmoidal) function because it successfully interprets the primary input and output relationship, and its differentiation power reduces the error to a minimum level. These features make it more applicable to the NN system in financial modeling.

Table 2

Dataset Distribution for (Training, Validation, and Testing)

Network Architecture (FF3F Model)	Network Architecture (FF5F Model)	Dataset Distribution	Hidden Layer (Neurons)
3-1-1 to 3-50-1	5-1-1 to 5-50-1	60-20-20	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	65-15-20	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	65-20-15	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	70-10-20	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	70-15-15	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	70-20-10	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	75-05-20	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	75-10-15	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	75-15-10	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	75-20-05	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	80-05-15	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	80-15-05	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	80-10-10	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	85-10-05	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	85-05-10	Neurons 1-50
3-1-1 to 3-50-1	5-1-1 to 5-50-1	90-05-05	Neurons 1-50

The most used sigmoidal function is a logistic function, and its curve is “S-shaped". Mathematically, it is represented as

$$S(x) = \frac{1}{1 + e^{-x}} \ldots\ldots\ldots\ldots (9)$$

The architecture of the NN system for the present study includes a hidden and output layer. The maximum limit of the hidden layer neurons is placed at 50. The programming code returns the optimum results for the neurons under each dataset. This system consists of a single target variable in the output layer.

In step 3, the training algorithm is programmed to select the optimal weights between the input variables (neurons). It enables the network to determine a global minimum error function with minimum computational power. Unless the model is habituated for local observation or fitting, the weight assignment in this way provides good generalization ability to the networks. (Ayub et al., 2020). The principal training algorithm is the backpropagation function with the Hessian matrix (Jacobian derivatives) and backpropagation based on gradient derivatives. The former functions use the Backpropagation algorithm of Levenberg–Marquardt. This algorithm is faster but consumes more memory. The latter also uses the scaled conjugate gradient (SCG) and quasi-Newton method (BFG). We select the LM, SCG, and BFG training methods to train the network.

There are two perspectives regarding the magnitude of training. The first is the possibility of being confined to a local minimum and the difficulty of attaining a global minimum. The second school of thought proposes the concept of a series of training-testing interruptions. We select the train-test interruption approach and limit the number of iterations to 1000. The performance metrics of mean squared error are used as a benchmark to compare the metric's forecasting performance. The mathematical representation of MSE is given below.

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(R_t - \hat{R}_t)^2 \dots\dots\dots\dots(10)$$

R_t and $\hat{R}_t$ represent the actual returns and forecasted returns, and N expresses the size of the testing dataset. The minimum MSE score is picked as the best lag point for each neural network architecture.

In step 4, we document the MATLAB instructions and run the algorithm over thirty (30) portfolios. The portfolios are sorted from high to low risk following the robust procedure adopted by (Fama & French, 2015). The results are reported for all the portfolios to examine the versatility of the ANN system for forecasting the time series of returns on various risk levels. This algorithmic rule predicts the portfolio returns at t+1 using the data from 1 to t. The first program returns the mean squared error between the actual and predicted values (returns) of the high, medium, and low beta portfolios for all the dataset neurons. This algorithm applies a simple one-step-ahead design.

The second set of instructions is based on rolling windows or look back windows scheme of 48 months rolling for forecasting purposes. This algorithm is also applied to high-, mid-, and low-beta portfolios based on the three-factor and five-factor models. Under this scheme, the MATLAB instructions read the first 48 monthly returns of the three and five input factors, forecast their returns one step ahead, compare them with the actual returns in the output layer, and calculate their output. The instructions roll forwards the data by one month by reading it from 2 to t+1 in the second loop. The procedure is repeated until the data are exhausted and the minimum error results are compiled for this series. This simulation is conducted for all the neuron and dataset combinations.

The presence of autocorrelation among the various forecasts is a potential problem due to the market's small size. Therefore, we test the forecasting accuracy of the estimated forecasting models by employing the well-known Diebold-Mariano test. The typical form of the Diebold-Mariano test is expressed by expressions 11 and 12.

$$\boldsymbol{S} = \boldsymbol{d}^{-}/\boldsymbol{s_d} \dots\dots (11)$$

$$\boldsymbol{d} = \boldsymbol{L_1} - \boldsymbol{L_2} \dots\dots (12)$$

In expression 11, d^{-} and s_d represent the mean and standard deviation of d, respectively. Similarly, in expression 12, the term d measures absolute or squared differences between the actual and forecasted values. The Diebold Mariano test assumes T-1 degrees of freedom, and further, it is based on the well-known Student's t-distribution. The null hypothesis under the Diebold-Mariano test is that forecast ''i'' includes all information contained in others, which means that the predictive accuracy of different forecasting models is equal. Therefore, accepting the null hypothesis would imply that accuracy differences do not exist for the estimated forecasting models. On the other hand, the rejection of the null hypothesis and acceptance of the alternative hypothesis would indicate significant differences in the forecasting accuracy of the estimated models.

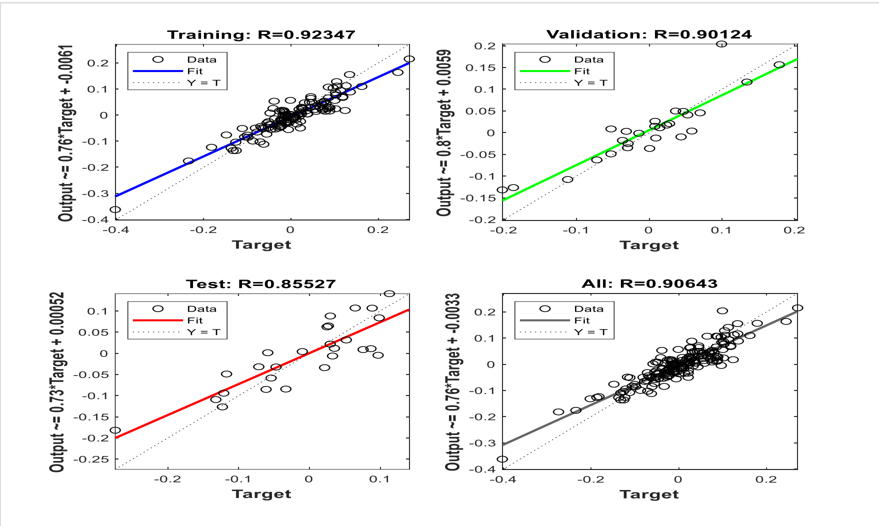
Results: Identifying the Optimal ANN Architecture using Equations (7) and (8).

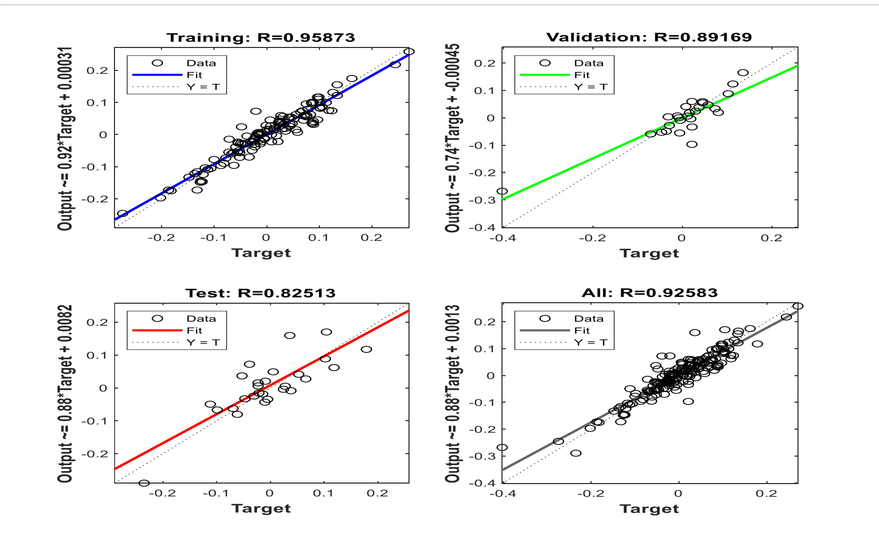
We estimated the dependence of the output variable (target portfolio returns) and independent variables. The regression diagram of the three- and five-factor models shows that the R-value of the three-factor model is approximately 90% (average of training, testing, and validation), while the five-factor model returns a 99% R-value, as displayed in Figures 1 and 2. The theory of asset pricing models states that market returns, size, value, profitability, and investment factors adequately explain portfolio returns, and our regression graph demonstrates this concept.

The neural network architectures based on equations (7) and (8) identified the best ANN system. The combination of three training functions and two asset pricing models resulted in 144,000 network models. These results were organized and separated on the risk level ($\beta's$) and summarized as high, mid, and low β portfolios in Table 3. This table reports the averages of the lowest mean squared error (MSE) and the optimal neurons of all the data combinations.

Figures 1 and 2

Regression Diagram (Fama and French 3 and 5 Factors Model)





Identifying the Optimal Forecasting Architecture for the (FF3F Model)

The three rows of Table 3 describe the three classes of Portfolios. The first six columns of this table report the best MSE statistics of the FF3F model under various training algorithms and the number of neurons.

We find that the lowest MSE score of 0.0042 is generated by most of the datasets using the LM training function at eight neurons for the three-factor model (lower part of Table 3).

The SCG and BFG functions converge with the lowest MSE scores of 0.0062 and 0.0060 at 16 and 23 nodes by most of our sample market data combinations. This analysis aims to identify the architecture that ensures the minimum difference between the actual and predicted values. We find that the minimization behavior of the dataset at 70% (training), 10% validation, and 20% testing at eight neurons under the LM method produces optimal results for all 50 neurons (Table A). We use this combination as the standard for the look-back windows under the three-factor model for high β portfolios in section (3.3).

The middle part of Table 3 reports the MSE score of mid-beta portfolios. The LM, SCG, and BFG algorithms return the best MSE results at 35, 19, and 23 neurons on the 60-20-20, 65-15-20, and 65-20-15 datasets, respectively, for the FF3F model. The corresponding MSE scores of these parameters, as reported in Table 3, are 0.005, 0.0074, and 0.0073. We select the 60-20-20 dataset and 35 neurons for the mid-beta portfolios as the best NN architecture under the LM algorithm for the rolling window scheme. The low-beta portfolios converged the best MSE results at 70-20-10, 65-20-15, and 60-20-20 with 13, 13, and 22 neurons, respectively, under the three training methods.

Table 3

MSE Score of Portfolios under Various Training Functions

		Fama and French 3 Factor Model						Fama and French 5 Factor Model					
	Data Set	MSE (LM)	ON	MSE (SCG)	ON	MSE (BFG)	ON	MSE (LM)	ON	MSE (SCG)	ON	MSE (BFG)	ON
MSE Score of Low-Beta Portfolios under Various Training Methods	60-20-20	0.0050	13	0.0072	13	0.0071	22	0.0036	13	0.0062	23	0.0060	44
	65-15-20	0.0050	13	0.0073	13	0.0072	22	0.0036	13	0.0063	23	0.0068	9
	65-20-15	0.0050	13	0.0071	13	0.0071	22	0.0036	13	0.0063	23	0.0065	23
	70-10-20	0.0051	18	0.0072	13	0.0072	22	0.0037	13	0.0063	23	0.0069	14
	70-15-15	0.0050	18	0.0075	13	0.0072	22	0.0036	13	0.0064	23	0.0062	33
	70-20-10	0.0050	13	0.0072	13	0.0071	22	0.0036	13	0.0066	9	0.0066	37
	75-05-20	0.0054	16	0.0072	13	0.0077	8	0.0036	13	0.0066	9	0.0068	24
	75-10-15	0.0052	13	0.0073	13	0.0073	22	0.0037	13	0.0068	23	0.0071	13
	75-15-10	0.0050	18	0.0075	16	0.0072	22	0.0037	13	0.0065	23	0.0063	44
	75-20-05	0.0050	13	0.0073	13	0.0072	22	0.0039	13	0.0063	23	0.0061	44
	80-05-15	0.0058	14	0.0073	13	0.0078	15	0.0037	13	0.0069	9	0.0074	29
	80-10-10	0.0053	18	0.0076	13	0.0073	22	0.0038	13	0.0066	9	0.0097	37
	80-15-05	0.0050	18	0.0072	13	0.0072	22	0.0039	13	0.0066	9	0.0076	8
	85-05-10	0.0061	17	0.0074	13	0.008	11	0.0039	13	0.0072	13	0.0075	24
	85-10-05	0.0054	16	0.0079	7	0.0074	22	0.004	13	0.0066	9	0.0076	5
	90-05-05	0.0068	20	0.0084	10	0.0082	3	0.0054	45	0.0078	9	0.0079	1
MSE Score of Mid-Beta Portfolios under Various Training Methods	60-20-20	0.0049	35	0.0075	12	0.0073	23	0.0024	25	0.006	17	0.006	26
	65-15-20	0.005	35	0.0074	19	0.0073	23	0.0034	50	0.006	17	0.006	26
	65-20-15	0.005	35	0.0074	19	0.0073	23	0.0034	50	0.0061	17	0.0061	26
	70-10-20	0.005	35	0.0075	18	0.0074	15	0.0035	50	0.0061	17	0.0061	26
	70-15-15	0.005	35	0.0075	18	0.0073	23	0.0035	50	0.007	17	0.007	37
	70-20-10	0.005	35	0.0074	18	0.0073	21	0.0034	50	0.007	17	0.007	4
	75-05-20	0.0054	35	0.0077	18	0.0075	15	0.0034	50	0.0069	17	0.0069	14
	75-10-15	0.0052	35	0.0075	18	0.0074	23	0.0035	30	0.007	17	0.007	15
	75-15-10	0.005	35	0.0075	18	0.0073	23	0.0036	30	0.0073	17	0.0073	27
	75-20-05	0.005	35	0.0074	18	0.0073	23	0.0041	50	0.007	17	0.007	27
	80-05-15	0.0054	35	0.0077	7	0.0076	15	0.0035	50	0.007	17	0.007	27
	80-10-10	0.0053	35	0.0076	9	0.0075	15	0.0037	50	0.007	17	0.007	8
	80-15-05	0.005	35	0.0074	18	0.0073	23	0.0042	17	0.0074	16	0.0074	2
	85-05-10	0.006	35	0.0077	18	0.0079	1	0.0037	50	0.0077	17	0.0077	1
	85-10-05	0.0053	35	0.0078	6	0.0075	15	0.0043	17	0.0067	17	0.0067	25
	90-05-05	0.007	7	0.0081	2	0.0083	3	0.0054	50	0.0078	11	0.0078	3

MSE Score of High Beta Portfolios under Various Training Methods

	Fama and French 3 Factor Model						Fama and French 5 Factor Model					
Data Set	MSE (LM)	ON	MSE (SCG)	ON	MSE (BFG)	ON	MSE (LM)	ON	MSE (SCG)	ON	MSE (BFG)	ON
60-20-20	0.0043	15	0.0062	16	0.006	23	0.0029	45	0.0053	21	0.0053	28
65-15-20	0.0042	8	0.0062	16	0.0062	26	0.003	45	0.0053	21	0.0057	18
65-20-15	0.0042	8	0.0062	16	0.0061	23	0.0029	45	0.0054	21	0.0057	18
70-10-20	0.0042	8	0.0063	16	0.0063	23	0.0031	40	0.0053	21	0.0058	37
70-15-15	0.0042	8	0.0062	16	0.0062	23	0.0031	45	0.0053	21	0.0059	19
70-20-10	0.0042	8	0.0062	16	0.0061	23	0.0029	45	0.006	5	0.0059	19
75-05-20	0.0048	14	0.0065	11	0.0065	11	0.0029	45	0.0058	14	0.006	33
75-10-15	0.0042	8	0.0064	16	0.0063	26	0.003	45	0.0056	16	0.0061	26
75-15-10	0.0042	8	0.0062	16	0.0064	7	0.0032	18	0.0056	16	0.0061	8
75-20-05	0.0042	8	0.0062	16	0.0066	11	0.0027	28	0.0061	14	0.0057	24
80-05-15	0.0049	18	0.0065	11	0.0066	11	0.0031	40	0.0058	14	0.0057	24
80-10-10	0.0042	8	0.0064	13	0.0062	23	0.0031	40	0.0054	21	0.0059	26
80-15-05	0.0042	8	0.0062	16	0.0064	26	0.0032	40	0.0062	14	0.0062	9
85-05-10	0.0052	14	0.0066	7	0.0093	26	0.0032	40	0.0056	23	0.0063	5
85-10-05	0.0045	25	0.0064	13	0.0065	26	0.0058	4	0.0063	1	0.006	17
90-05-05	0.0056	14	0.0071	15	0.0069	2	0.0055	16	0.0056	1	0.0068	4

ON: Optimal Neuron

Table 3 reports the lowest MSEs of 0.005, 0.0071, and 0.0071 for the abovementioned NN design. The low-beta portfolios utilize the 70-20-10 dataset with 13 hidden nodes and the LM method for the rolling window scheme.

This analysis enables us to conclude that the backpropagation training function (the Jacobian derivatives) with the LM algorithm produces the lowest forecasting results more efficiently in the stock markets than other training functions. It is a significant finding of our analysis, and investors can apply these optimal architectures of ANN models to forecast their portfolio returns depending on the risk level. The random selection of the stocks and variables, on the other hand, under the ANN system may develop a good academic exercise without having practical implications in the stock markets. Investment decisions are mainly based on portfolio formation and management, while the random selection of firms and variables is time-consuming and lacks the ability to generalize in global markets.

Efficient ANN Architecture Forecasting (FF5F Model)

The five-factor model nests the three-factor model and contains the additional factors of investments and profitability. The last six columns of Table 3 present the MSE statistics of the FF5F model. The best MSE value for high-risk portfolios converges at the 75-20-05, 60-20-20, and 60-20-20 datasets. Table 3 reports that the MSE scores of these data combinations at 28, 21, and 28 neurons are 0.0027, 0.0053, and 0.0053, respectively. The high-beta portfolios ensure the best ANN system at the 75-20-05 data combination on the LM method with 28 neurons. This combination is further used for the rolling window scheme for this class of portfolios. A comparative analysis of this finding with the three factors of high beta portfolios shows that the additional factors of investment and profitability have reduced the forecasting error of our proposed ANN model by 44%. The five-factor model has demonstrated more accurate prediction performance than the previous model, and the finding suggests that the five-factor CAPM has significant relevance in magnifying the returns of the portfolios containing high-risk stocks for investors in this frontier market. It is interesting to note here that some recent studies, for example, Racicot and Rentz (2016), find all other factors of the FF5F model except market returns to be irrelevant.

The forecasting performance of the NN system in the case of the medium-risk portfolios is identical in terms of the data distribution for the three-factor and five-factor models but different in the number of neurons. The identical dataset is 60-20-20, and the number of optimal neurons is 35 and 25 on the LM algorithm. Table 3 reports the lowest MSE score of 0.0024 for the mid-beta portfolios for the FF5F model. This error is 50% lower than the error reported by the three-factor model for the same risk level.

The low beta results of the three- and five-factor models are also identical in terms of the dataset and neurons. The dataset 60-20-20 at 13 neurons converges to the minimum MSE of 0.0036 for the FF5F model. It shows a 35% error reduction compared to the FF3F model. These datasets and the optimal neurons are further employed for the look-back windows. The comparison of the MSE statistics of low- and high-beta portfolios signifies that the market compensates more for high-risk portfolios than for low-beta portfolios. These findings provide evidence that neural networks demonstrate high forecasting accuracy when processed with the variables of asset pricing models. We find more than 90% accuracy in our proposed hybridized system. This accuracy level contrasts sharply with the previous levels, wherein 80% to 85% forecasting accuracy is reported for ANN systems (Pyo et al., 2017). The minimum errors reported for the three types of portfolios can enable investors to earn excess market returns.

Actual vs. Predicted Returns Under Rolling Window Scheme and NN System

In this section, we further examine the optimal NN system, identified in section 3.1, and apply a look-back window scheme with a 48-monthly estimation window of returns and rolled forwards monthly. The scheme is applied to six ANN architectures only for the optimal training function of Levenberg–Marquardt. We obtain 132 networks for each portfolio, representing the same number of forecasting statistics for each architecture. Three thousand nine hundred sixty monthly predictions are generated for each asset pricing model, and these returns are converted into annualized returns.

Table 4

Actual Vs. Predicted Portfolio Return under Levenberg–Marquardt training Function

			Fama and French 3 Factor Model				Fama and French 5 Factor Model			
	Year	Actual Returns	Predicted Return%	MSE	Dataset	ON	Predicted Returns %	MSE	Data Set	ON
Actual vs. Predicted Returns (High Beta Portfolios)	2005	30.09	29.47	0.0062	70-10-20	8	29.96	0.0013	75-05-20	28
	2006	22.93	22.24	0.0069	70-10-20	8	22.71	0.0022	75-05-20	28
	2007	-1.9	-2.48	0.0058	70-10-20	8	-2.19	0.0029	75-05-20	28
	2008	16.83	16.32	0.0051	70-10-20	8	16.46	0.0037	75-05-20	28
	2009	-54.44	-54.99	0.0055	70-10-20	8	-54.89	0.0045	75-05-20	28
	2010	-15.95	-17.3	0.0135	70-10-20	8	-16.49	0.0054	75-05-20	28
	2011	-19.19	-20.43	0.0124	70-10-20	8	-19.84	0.0065	75-05-20	28
	2012	-11	-12.27	0.0127	70-10-20	8	-11.78	0.0078	75-05-20	28
	2013	12.03	11.1	0.0093	70-10-20	8	11.25	0.0078	75-05-20	28
	2014	20.35	19.99	0.0036	70-10-20	8	20.22	0.0013	75-05-20	28
	2015	21.04	20.69	0.0035	70-10-20	8	20.8	0.0024	75-05-20	28
Actual vs. Predicted Returns (Mid Beta Portfolios)	2005	32.26	31.39	0.0087	60-20-20	35	31.98	0.0028	60-20-20	25
	2006	22.24	21.3	0.0094	60-20-20	35	21.84	0.004	60-20-20	25
	2007	-14.2	-14.93	0.0073	60-20-20	35	-14.91	0.0071	60-20-20	25
	2008	29.95	29.39	0.0056	60-20-20	35	29.31	0.0064	60-20-20	25
	2009	-51.85	-52.47	0.0062	60-20-20	35	-52.59	0.0074	60-20-20	25
	2010	-25.04	-26.55	0.0151	60-20-20	35	-25.27	0.0023	60-20-20	25
	2011	-15.67	-17.1	0.0143	60-20-20	35	-15.86	0.0019	60-20-20	25
	2012	-35.03	-36.68	0.0165	60-20-20	35	-35.26	0.0023	60-20-20	25
	2013	55.68	54.44	0.0124	60-20-20	35	55.44	0.0024	60-20-20	25
	2014	15.29	14.76	0.0053	60-20-20	35	14.77	0.0052	60-20-20	25
	2015	40.56	39.98	0.0058	60-20-20	35	39.98	0.0058	60-20-20	25
Actual vs. Predicted Returns (Low Beta Portfolios)	2005	55.52	54.36	0.0116	70-20-10	8	54.86	0.0066	70-20-10	13
	2006	23.75	22.7	0.0105	70-20-10	8	22.9	0.0085	70-20-10	13
	2007	-18.05	-18.73	0.0068	70-20-10	8	-18.69	0.0064	70-20-10	13
	2008	41.37	40.84	0.0053	70-20-10	8	40.84	0.0053	70-20-10	13
	2009	-49.03	-49.57	0.0054	70-20-10	8	-49.63	0.006	70-20-10	13
	2010	-28.37	-29.35	0.0098	70-20-10	8	-29.43	0.0106	70-20-10	13
	2011	-22.17	-23.28	0.0111	70-20-10	8	-23.26	0.0109	70-20-10	13
	2012	-25.45	-26.63	0.0118	70-20-10	8	-26.57	0.0112	70-20-10	13
	2013	70.56	69.7	0.0086	70-20-10	8	69.55	0.0101	70-20-10	13
	2014	12.09	11.52	0.0057	70-20-10	8	11.58	0.0051	70-20-10	13
	2015	30.53	29.96	0.0057	70-20-10	8	29.99	0.0054	70-20-10	13

The predicted annual returns are compared with actual values year by year. The use of 4 years of rolling is a standard in financial research, as noted by (Ayub et al., 2015). The results are presented in Table 4.

Table 4 displays the actual and predicted returns for eleven years. The analysis of this table reveals that the predicted returns are in the range of 29.47% and -54.99 using the factors of the FF3F model, while the FF5F model returns 29.96% and -54.89% predicted returns% for high beta portfolios. The range of actual returns for HB portfolios is 30.09% and -54.44%. This shows that the ANN configuration of the FF5F model demonstrates more accurate forecasting than the three-factor model. The medium beta portfolios converge 54.44% and -54.99% maximum and minimum predicted returns, while the low beta portfolios generate 69.70% and -49.57% predicted returns for the FF3F model. The actual returns of medium- and low-risk portfolios range between 55.68% and -54.44%. The ANNs under the FF5F model produce 55.44% and -52.59% results for medium beta and 69.55%-49.63% predicted values for low beta returns.

The analysis further shows that the system's NN-generated returns closely follow the actual portfolio values most of the time. The ANN models have successfully captured the sequence of monthly returns and exhibit a high ability to identify the rise and fall of this frontier market, with some exceptions. It is a significant finding, suggesting that various asset pricing models and artificial neural networks accurately predict the direction of the market along with a closer depiction of returns.

The analysis of Table 4 further presents the enhanced predictability of the networks on selected portfolios. This table shows that the predicted values closely match the market in most instances on both pricing models. However, less risky portfolios exhibit wide variation in most of the intervals; investors have lower chances of magnified returns in low-risk stocks. The theoretical foundation of the five-factor CAPM postulates that the model explains the average returns of low-earning firms more accurately.

Testing the Forecasting Accuracy of FF3F and FF5F Models

The results for examining the forecasting accuracy based on the Diebold-Mariano test are presented in Table 5. According to the results, the null hypothesis of no differences in forecasting accuracy or equal predictive accuracy of models is rejected at the 1 percent level. This was the case for low, medium, and high β portfolios for all the estimated forecasting models. Therefore, the alternative hypothesis shall be accepted, and it can be concluded that the forecasting accuracy under the five-factor model is robust and more accurate than that under the three-factor model.

Table 5

Diebold-Mariano Forecasting Accura+3cy Test

Forecast Evaluation	F-stat	F-probability
FF3FLB VS FF5FLB	82.800***	0
FF3FMB VS FF5FMB	43.356***	0
FF3FHB VS FF5FHB	55.764***	0

*The Diebold-Mariano Test is carried out in Eviews 9.0. where *** indicates significance at the 1 percent level.*

Conclusions and Recommendations

This article examines the forecasting performance of various asset pricing models in the presence of artificial neural networks, and the prime purpose is to identify the model that provides the optimum depiction of portfolio returns in Pakistan's equity market. ANN is used as the tool of principal measure, and three categories of Portfolios, i.e., high-, mid-, and low-risk portfolios, are constructed. ANN testing utilizes wide-ranging parameters, and applying the rolling scheme to portfolio returns establishes the strength and flexibility of the proposed ANN models.

The Fama and French three- and five-factor models are utilized to evaluate the simulated returns of Portfolios under differing training functions, datasets, and a wide range of neurons of the ANN models. This hybridization of ANN and asset pricing models affirm that the composite factors can predict portfolio returns and the accuracy rate is more than 90 percent. A major finding of our experimentation reveals that different architectures of ANNs and the Levenberg Marquardt algorithm converge the best-forecasted returns for high-, mid-, and low-beta portfolios, while previous studies (Cao et al., 2005; Qiu & Song, 2016) suggested a uniform ANN architecture for stock market prediction.

The comparative analysis of the three- and five-factor models reveals that the additional factors of the five-factor models have significantly improved the forecasting performance of our ANN system. The ANN-FF5F architecture has reduced the average forecasting error by 43% for the low, mid, and high beta portfolios. A rolling window scheme was used to examine the stability of the optimal ANN structures (48 months). This experimentation shows that the proposed network system demonstrates resilience to accommodate the market's directional movements, showing the models' stability. This is a significant finding, and stock traders can apply these models to forecast stocks of varying risk levels, thus avoiding potential losses and increasing the return on investment. The low-risk portfolios, however, witness widespread variation in returns; the low-risk Portfolios have lower chances of accurate prediction.

The Diebold Mariano forecasting accuracy test results reveal that the forecasting accuracy reported by various architectures and asset pricing models is significantly different. This test verifies that the five-factor model under the ANN architecture is the optimal combination for all classes of portfolios.

Researchers have tested the forecasting strength of ANNs in many disciplines, including stock markets. Investors can expect the opportunity for excess returns by employing the learning capability of ANN in the data-rich markets of stock exchanges. This research opens more avenues for further experimentation. Future finance researchers need to consider the application of deep neural networks in the clustering and classification of stock market returns. The use of ANN in the initial stage of Portfolio Formation and a comparative analysis with other linear and nonlinear forecasting models will help determine the robustness of ANN. Applying the cross-validation technique for datasets and regression-based neural networks to the theory of asset pricing is another vital proposition for future research.

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